of real numbers. We denote by $\mathbb{R}^n$ the set of all such n-tuples. For example, if a company uses n different ingredients in making a food product, c_i is the cost per unit of the ith ingredient, and x_i units of the ith ingredient are used, then the total cost C of the ingredients is a function of the n variables $x_1, x_2, \ldots, x_n$:

$$C = f(x_1, x_2, \ldots, x_n) = c_1x_1 + c_2x_2 + \cdots + c_nx_n \tag{3}$$

The function f is a real-valued function whose domain is a subset of $\mathbb{R}^n$. Sometimes we use vector notation to write such functions more compactly: If $\mathbf{x} = \langle x_1, x_2, \ldots, x_n \rangle$, we often write $f(\mathbf{x})$ in place of $f(x_1, x_2, \ldots, x_n)$. With this notation we can rewrite the function defined in Equation 3 as

$$f(\mathbf{x}) = \mathbf{c} \cdot \mathbf{x}$$

where $\mathbf{c} = \langle c_1, c_2, \ldots, c_n \rangle$ and $\mathbf{c} \cdot \mathbf{x}$ denotes the dot product of the vectors $\mathbf{c}$ and $\mathbf{x}$ in V_n.

In view of the one-to-one correspondence between points $(x_1, x_2, \ldots, x_n)$ in $\mathbb{R}^n$ and their position vectors $\mathbf{x} = \langle x_1, x_2, \ldots, x_n \rangle$ in V_n, we have three ways of looking at a function f defined on a subset of $\mathbb{R}^n$:

1. As a function of n real variables $x_1, x_2, \ldots, x_n$
2. As a function of a single point variable $(x_1, x_2, \ldots, x_n)$
3. As a function of a single vector variable $\mathbf{x} = \langle x_1, x_2, \ldots, x_n \rangle$

We will see that all three points of view are useful.

14.1 Exercises

1. If $f(x, y) = x^2y/(2x - y^2)$, find
(a) $f(1, 3)$ (b) $f(-2, -1)$
(c) $f(x + h, y)$ (d) $f(x, x)$

2. If $g(x, y) = x \sin y + y \sin x$, find
(a) $g(\pi, 0)$ (b) $g(\pi/2, \pi/4)$
(c) $g(0, y)$ (d) $g(x, y + h)$

3. Let $g(x, y) = x^2 \ln(x + y)$.
(a) Evaluate $g(3, 1)$.
(b) Find and sketch the domain of g.
(c) Find the range of g.

4. Let $h(x, y) = e^{\sqrt{y - x^2}}$.
(a) Evaluate $h(-2, 5)$.
(b) Find and sketch the domain of h.
(c) Find the range of h.

5. Let $F(x, y, z) = \sqrt{y} - \sqrt{x - 2z}$.
(a) Evaluate $F(3, 4, 1)$.
(b) Find and describe the domain of F.

6. Let $f(x, y, z) = \ln\left(z - \sqrt{x^2 + y^2}\right)$.
(a) Evaluate $f(4, -3, 6)$.
(b) Find and describe the domain of f.

7–16 Find and sketch the domain of the function.

7. $f(x, y) = \sqrt{x - 2} + \sqrt{y - 1}$

8. $f(x, y) = \sqrt[4]{x - 3y}$

9. $q(x, y) = \sqrt{x} + \sqrt{4 - 4x^2 - y^2}$

10. $g(x, y) = \ln(x^2 + y^2 - 9)$

11. $g(x, y) = \dfrac{x - y}{x + y}$

12. $g(x, y) = \dfrac{\ln(2 - x)}{1 - x^2 - y^2}$

13. $p(x, y) = \dfrac{\sqrt{xy}}{x + 1}$

14. $f(x, y) = \sin^{-1}(x + y)$

15. $f(x, y, z) = \sqrt{4 - x^2} + \sqrt{9 - y^2} + \sqrt{1 - z^2}$

16. $f(x, y, z) = \ln(16 - 4x^2 - 4y^2 - z^2)$

17. A model for the surface area of a human body is given by the function

$$S = f(w, h) = 0.1091w^{0.425}h^{0.725}$$

where w is the weight (in pounds), h is the height (in inches), and S is measured in square feet.
(a) Find $f(160, 70)$ and interpret it.
(b) What is your own surface area?

18. A manufacturer has modeled its yearly production function P (the monetary value of its entire production in millions of dollars) as a Cobb-Douglas function

$$P(L, K) = 1.47L^{0.65}K^{0.35}$$

where L is the number of labor hours (in thousands) and K is the invested capital (in millions of dollars). Find $P(120, 20)$ and interpret it.

19. In Example 3 we considered the function $W = f(T, v)$, where W is the wind-chill index, T is the actual temperature, and v is the wind speed. A numerical representation is given in Table 1.

(a) What is the value of $f(-15, 40)$? What is its meaning?
(b) Describe in words the meaning of the question "For what value of v is $f(-20, v) = -30$?" Then answer the question.
(c) Describe in words the meaning of the question "For what value of T is $f(T, 20) = -49$?" Then answer the question.
(d) What is the meaning of the function $W = f(-5, v)$? Describe the behavior of this function.
(e) What is the meaning of the function $W = f(T, 50)$? Describe the behavior of this function.

20. The *temperature-humidity index* I (or humidex, for short) is the perceived air temperature when the actual temperature is T and the relative humidity is h, so we can write $I = f(T, h)$. The following table of values of I is an excerpt from a table compiled by the National Oceanic & Atmospheric Administration.

Table 3 Apparent temperature as a function of temperature and humidity

Relative humidity (%) — rows: Actual temperature (°F)

T \ h	20	30	40	50	60	70
80	77	78	79	81	82	83
85	82	84	86	88	90	93
90	87	90	93	96	100	106
95	93	96	101	107	114	124
100	99	104	110	120	132	144

(a) What is the value of $f(95, 70)$? What is its meaning?
(b) For what value of h is $f(90, h) = 100$?
(c) For what value of T is $f(T, 50) = 88$?
(d) What are the meanings of the functions $I = f(80, h)$ and $I = f(100, h)$? Compare the behavior of these two functions of h.

21. The wave heights h in the open sea depend on the speed v of the wind and the length of time t that the wind has been blowing at that speed. Values of the function $h = f(v, t)$ are recorded in feet in Table 4.

(a) What is the value of $f(40, 15)$? What is its meaning?
(b) What is the meaning of the function $h = f(30, t)$? Describe the behavior of this function.
(c) What is the meaning of the function $h = f(v, 30)$? Describe the behavior of this function.

Table 4 Wave height as a function of wind speed and duration

Duration (hours) — rows: Wind speed (knots)

v \ t	5	10	15	20	30	40	50
10	2	2	2	2	2	2	2
15	4	4	5	5	5	5	5
20	5	7	8	8	9	9	9
30	9	13	16	17	18	19	19
40	14	21	25	28	31	33	33
50	19	29	36	40	45	48	50
60	24	37	47	54	62	67	69

22. A company makes three sizes of cardboard boxes: small, medium, and large. It costs \$2.50 to make a small box, \$4.00 for a medium box, and \$4.50 for a large box. Fixed costs are \$8000.

(a) Express the cost of making x small boxes, y medium boxes, and z large boxes as a function of three variables: $C = f(x, y, z)$.
(b) Find $f(3000, 5000, 4000)$ and interpret it.
(c) What is the domain of f?

23–31 Sketch the graph of the function.

23. $f(x, y) = y$

24. $f(x, y) = x^2$

25. $f(x, y) = 10 - 4x - 5y$

26. $f(x, y) = \cos y$

27. $f(x, y) = \sin x$

28. $f(x, y) = 2 - x^2 - y^2$

29. $f(x, y) = x^2 + 4y^2 + 1$

30. $f(x, y) = \sqrt{4x^2 + y^2}$

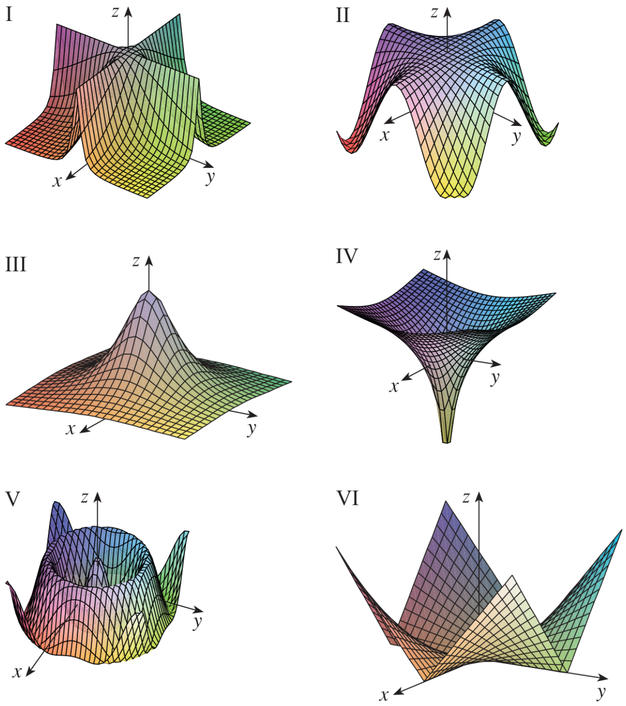
31. $f(x, y) = \sqrt{4 - 4x^2 - y^2}$

32. Match the function with its graph (labeled I–VI). Give reasons for your choices.

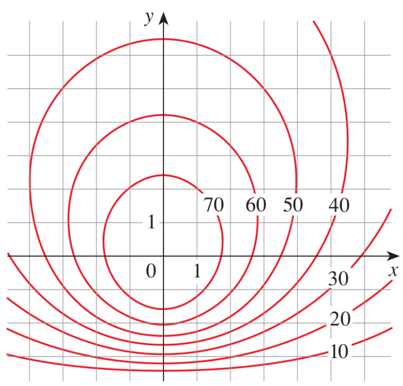
(a) $f(x, y) = \dfrac{1}{1 + x^2 + y^2}$ (b) $f(x, y) = \dfrac{1}{1 + x^2y^2}$

(c) $f(x, y) = \ln(x^2 + y^2)$ (d) $f(x, y) = \cos\sqrt{x^2 + y^2}$

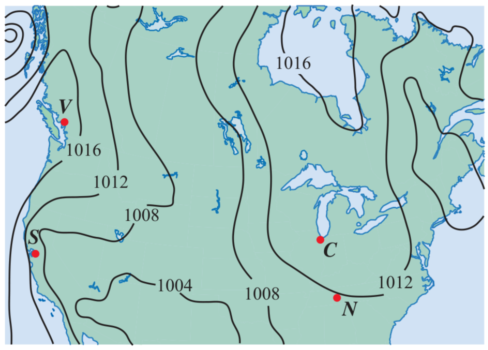
(e) $f(x, y) = |xy|$ (f) $f(x, y) = \cos(xy)$



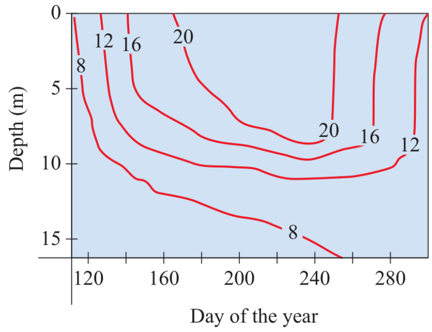
33. A contour map for a function f is shown. Use it to estimate the values of $f(-3, 3)$ and $f(3, -2)$. What can you say about the shape of the graph?



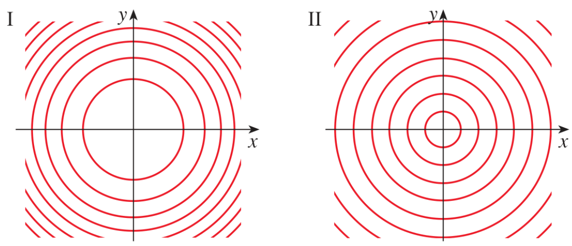
34. Shown is a contour map of atmospheric pressure in North America on a particular day. On the level curves (isobars) the pressure is indicated in millibars (mb).
(a) Estimate the pressure at C (Chicago), N (Nashville), S (San Francisco), and V (Vancouver).
(b) At which of these locations were the winds strongest? (See the discussion preceding Example 9.)



35. Level curves (isothermals) are shown for the typical water temperature (in °C) in Long Lake (Minnesota) as a function of depth and time of year. Estimate the temperature in the lake on June 9 (day 160) at a depth of 10 m and on June 29 (day 180) at a depth of 5 m.

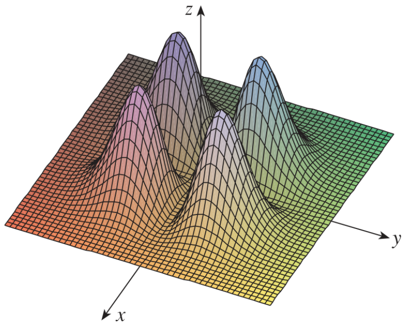


36. Two contour maps are shown. One is for a function f whose graph is a cone. The other is for a function g whose graph is a paraboloid. Which is which, and why?



37. Locate the points A and B on the map of Lonesome Mountain (Figure 12). How would you describe the terrain near A? Near B?

38. Make a rough sketch of a contour map for the function whose graph is shown.



39. The *body mass index* (BMI) of a person is defined by

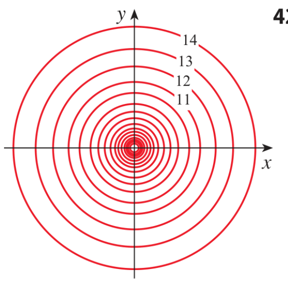
$$B(m, h) = \frac{m}{h^2}$$

where m is the person's mass (in kilograms) and h is the person's height (in meters). Draw the level curves $B(m, h) = 18.5$, $B(m, h) = 25$, $B(m, h) = 30$, and $B(m, h) = 40$. A rough guideline is that a person is underweight if the BMI is less than 18.5; optimal if the BMI lies between 18.5 and 25; overweight if the BMI lies between 25 and 30; and obese if the BMI exceeds 30. Shade the region corresponding to optimal BMI. Does someone who weighs 62 kg and is 152 cm tall fall into the optimal category?

40. The body mass index is defined in Exercise 39. Draw the level curve of this function corresponding to someone who is 200 cm tall and weighs 80 kg. Find the weights and heights of two other people with that same level curve.

41–44 A contour map of a function is shown. Use it to make a rough sketch of the graph of f.

41.



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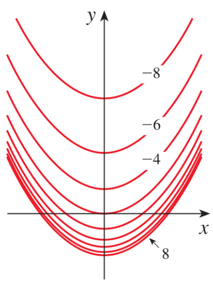

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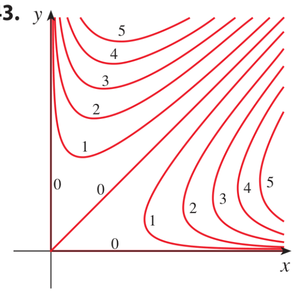

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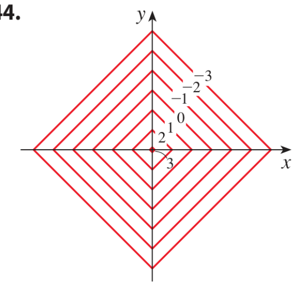


45–52 Draw a contour map of the function showing several level curves.

45. $f(x, y) = x^2 - y^2$

46. $f(x, y) = xy$

47. $f(x, y) = \sqrt{x} + y$

48. $f(x, y) = \ln(x^2 + 4y^2)$

49. $f(x, y) = ye^x$

50. $f(x, y) = y - \arctan x$

51. $f(x, y) = \sqrt[3]{x^2 + y^2}$

52. $f(x, y) = y/(x^2 + y^2)$

53–54 Sketch both a contour map and a graph of the given function and compare them.

53. $f(x, y) = x^2 + 9y^2$

54. $f(x, y) = \sqrt{36 - 9x^2 - 4y^2}$

55. A thin metal plate, located in the xy-plane, has temperature $T(x, y)$ at the point (x, y). Sketch some level curves (isothermals) if the temperature function is given by

$$T(x, y) = \frac{100}{1 + x^2 + 2y^2}$$

56. If $V(x, y)$ is the electric potential at a point (x, y) in the xy-plane, then the level curves of V are called *equipotential curves* because at all points on such a curve the electric potential is the same. Sketch some equipotential curves if $V(x, y) = c/\sqrt{r^2 - x^2 - y^2}$, where c is a positive constant.

57–60 Graph the function using various domains and viewpoints. If your software also produces level curves, then plot some contour lines of the same function and compare with the graph.

57. $f(x, y) = xy^2 - x^3$ (monkey saddle)

58. $f(x, y) = xy^3 - yx^3$ (dog saddle)

59. $f(x, y) = e^{-(x^2+y^2)/3}(\sin(x^2) + \cos(y^2))$

60. $f(x, y) = \cos x \cos y$

61–66 Match the function (a) with its graph (labeled A–F below) and (b) with its contour map (labeled I–VI). Give reasons for your choices.

61. $z = \sin(xy)$

62. $z = e^x \cos y$

63. $z = \sin(x - y)$

64. $z = \sin x - \sin y$

65. $z = (1 - x^2)(1 - y^2)$

66. $z = \dfrac{x - y}{1 + x^2 + y^2}$

67–70 Describe the level surfaces of the function.

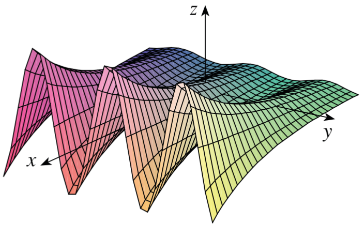
67. $f(x, y, z) = 2y - z + 1$

68. $g(x, y, z) = x + y^2 - z^2$

69. $g(x, y, z) = x^2 + y^2 - z^2$

70. $f(x, y, z) = x^2 + 2y^2 + 3z^2$

A



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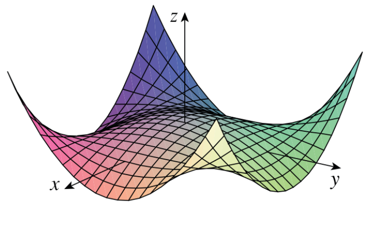

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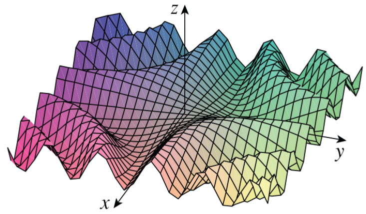

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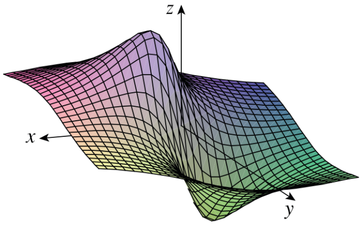

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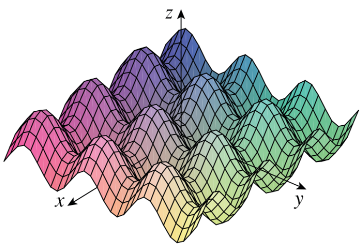

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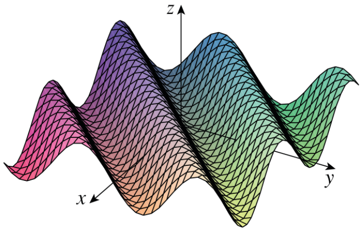

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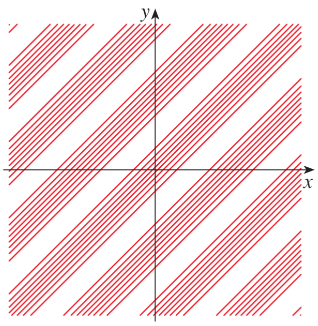

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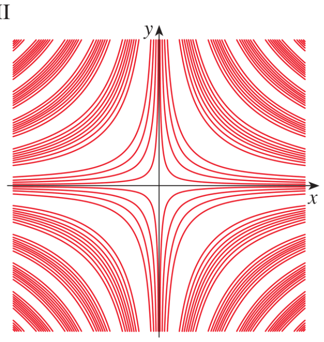
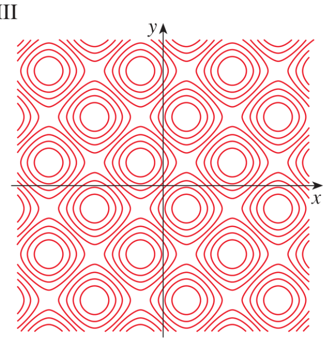


III



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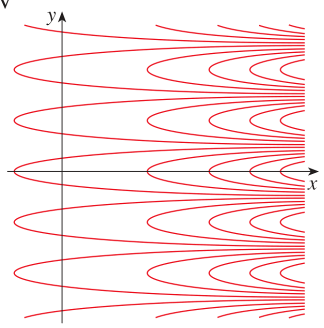

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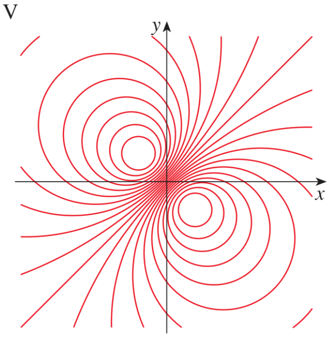

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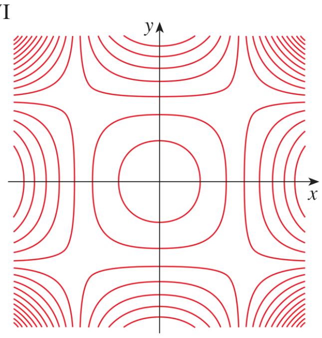

71–72 Describe how the graph of g is obtained from the graph of f.

71. (a) $g(x, y) = f(x, y) + 2$
(b) $g(x, y) = 2f(x, y)$
(c) $g(x, y) = -f(x, y)$
(d) $g(x, y) = 2 - f(x, y)$

72. (a) $g(x, y) = f(x - 2, y)$
(b) $g(x, y) = f(x, y + 2)$
(c) $g(x, y) = f(x + 3, y - 4)$

73–74 Graph the function using various domains and viewpoints that give good views of the "peaks and valleys." Would you say the function has a maximum value? Can you identify any points on the graph that you might consider to be "local maximum points"? What about "local minimum points"?

73. $f(x, y) = 3x - x^4 - 4y^2 - 10xy$

74. $f(x, y) = xye^{-x^2-y^2}$

75–76 Graph the function using various domains and viewpoints. Comment on the limiting behavior of the function. What happens as both x and y become large? What happens as (x, y) approaches the origin?

75. $f(x, y) = \dfrac{x + y}{x^2 + y^2}$ **76.** $f(x, y) = \dfrac{xy}{x^2 + y^2}$

77. Investigate the family of functions $f(x, y) = e^{cx^2+y^2}$. How does the shape of the graph depend on c?

78. Investigate the family of surfaces

$$z = (ax^2 + by^2)e^{-x^2-y^2}$$

How does the shape of the graph depend on the numbers a and b?

79. Investigate the family of surfaces $z = x^2 + y^2 + cxy$. In particular, you should determine the transitional values of c for which the surface changes from one type of quadric surface to another.

80. Graph the functions

$$f(x, y) = \sqrt{x^2 + y^2}$$

$$f(x, y) = e^{\sqrt{x^2+y^2}}$$

$$f(x, y) = \ln\sqrt{x^2 + y^2}$$

$$f(x, y) = \sin\left(\sqrt{x^2 + y^2}\right)$$

and $$f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$$

In general, if g is a function of one variable, how is the graph of

$$f(x, y) = g\left(\sqrt{x^2 + y^2}\right)$$

obtained from the graph of g?

81. (a) Show that, by taking logarithms, the general Cobb-Douglas function $P = bL^{\alpha}K^{1-\alpha}$ can be expressed as

$$\ln \frac{P}{K} = \ln b + \alpha \ln \frac{L}{K}$$

T (b) If we let $x = \ln(L/K)$ and $y = \ln(P/K)$, the equation in part (a) becomes the linear equation $y = \alpha x + \ln b$. Use Table 2 (in Example 4) to make a table of values of $\ln(L/K)$ and $\ln(P/K)$ for the years 1899–1922. Then find the least squares regression line through the points $(\ln(L/K), \ln(P/K))$.
(c) Deduce that the Cobb-Douglas production function is $P = 1.01L^{0.75}K^{0.25}$.

14.2 Limits and Continuity

Limits of Functions of Two Variables

Let's compare the behavior of the functions

$$f(x, y) = \frac{\sin(x^2 + y^2)}{x^2 + y^2} \qquad \text{and} \qquad g(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$$

as x and y both approach 0 [and therefore the point (x, y) approaches the origin].

14.2 Exercises

1. Suppose that $\lim_{(x, y) \to (3, 1)} f(x, y) = 6$. What can you say about the value of $f(3, 1)$? What if f is continuous?

2. Explain why each function is continuous or discontinuous.
(a) The outdoor temperature as a function of longitude, latitude, and time
(b) Elevation (height above sea level) as a function of longitude, latitude, and time
(c) The cost of a taxi ride as a function of distance traveled and time

3–4 Use a table of numerical values of $f(x, y)$ for (x, y) near the origin to make a conjecture about the value of the limit of $f(x, y)$ as $(x, y) \to (0, 0)$. Then explain why your guess is correct.

3. $f(x, y) = \dfrac{x^2y^3 + x^3y^2 - 5}{2 - xy}$ **4.** $f(x, y) = \dfrac{2xy}{x^2 + 2y^2}$

5–12 Find the limit.

5. $\lim_{(x, y) \to (3, 2)} (x^2y^3 - 4y^2)$

6. $\lim_{(x, y) \to (5, -2)} (x^2y + 3xy^2 + 4)$

7. $\lim_{(x, y) \to (-3, 1)} \dfrac{x^2y - xy^3}{x - y + 2}$ **8.** $\lim_{(x, y) \to (2, -1)} \dfrac{x^2y + xy^2}{x^2 - y^2}$

9. $\lim_{(x, y) \to (\pi, \pi/2)} y \sin(x - y)$ **10.** $\lim_{(x, y) \to (3, 2)} e^{\sqrt{2x - y}}$

11. $\lim_{(x, y) \to (1, 1)} \left(\dfrac{x^2y^3 - x^3y^2}{x^2 - y^2} \right)$ **12.** $\lim_{(x, y) \to (\pi, \pi/2)} \dfrac{\cos y - \sin 2y}{\cos x \cos y}$

13–18 Show that the limit does not exist.

13. $\lim_{(x, y) \to (0, 0)} \dfrac{y^2}{x^2 + y^2}$ **14.** $\lim_{(x, y) \to (0, 0)} \dfrac{2xy}{x^2 + 3y^2}$

15. $\lim_{(x, y) \to (0, 0)} \dfrac{(x + y)^2}{x^2 + y^2}$ **16.** $\lim_{(x, y) \to (0, 0)} \dfrac{x^2 + xy^2}{x^4 + y^2}$

17. $\lim_{(x, y) \to (0, 0)} \dfrac{y^2 \sin^2 x}{x^4 + y^4}$ **18.** $\lim_{(x, y) \to (1, 1)} \dfrac{y - x}{1 - y + \ln x}$

19–30 Find the limit, if it exists, or show that the limit does not exist.

19. $\lim_{(x, y) \to (-1, -2)} (x^2y - xy^2 + 3)^3$

20. $\lim_{(x, y) \to (\pi, 1/2)} e^{xy} \sin xy$

21. $\lim_{(x, y) \to (2, 3)} \dfrac{3x - 2y}{4x^2 - y^2}$ **22.** $\lim_{(x, y) \to (1, 2)} \dfrac{2x - y}{4x^2 - y^2}$

23. $\lim_{(x, y) \to (0, 0)} \dfrac{xy^2 \cos y}{x^2 + y^4}$ **24.** $\lim_{(x, y) \to (0, 0)} \dfrac{x^3 - y^3}{x^2 + xy + y^2}$

25. $\lim_{(x, y) \to (0, 0)} \dfrac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}$

26. $\lim_{(x, y) \to (0, 0)} \dfrac{xy^4}{x^2 + y^8}$

27. $\lim_{(x, y, z) \to (6, 1, -2)} \sqrt{x + z} \cos(\pi y)$

28. $\lim_{(x, y, z) \to (0, 0, 0)} \dfrac{xy + yz}{x^2 + y^2 + z^2}$

29. $\lim_{(x, y, z) \to (0, 0, 0)} \dfrac{xy + yz^2 + xz^2}{x^2 + y^2 + z^4}$

30. $\lim_{(x, y, z) \to (0, 0, 0)} \dfrac{x^4 + y^2 + z^3}{x^4 + 2y^2 + z}$

31–34 Use the Squeeze Theorem to find the limit.

31. $\lim_{(x, y) \to (0, 0)} xy \sin \dfrac{1}{x^2 + y^2}$ **32.** $\lim_{(x, y) \to (0, 0)} \dfrac{xy}{\sqrt{x^2 + y^2}}$

33. $\lim_{(x, y) \to (0, 0)} \dfrac{xy^4}{x^4 + y^4}$

34. $\lim_{(x, y, z) \to (0, 0, 0)} \dfrac{x^2y^2z^2}{x^2 + y^2 + z^2}$

35–36 Use a graph of the function to explain why the limit does not exist.

35. $\lim_{(x, y) \to (0, 0)} \dfrac{2x^2 + 3xy + 4y^2}{3x^2 + 5y^2}$ **36.** $\lim_{(x, y) \to (0, 0)} \dfrac{xy^3}{x^2 + y^6}$

37–38 Find $h(x, y) = g(f(x, y))$ and the set of points at which h is continuous.

37. $g(t) = t^2 + \sqrt{t}, \quad f(x, y) = 2x + 3y - 6$

38. $g(t) = t + \ln t, \quad f(x, y) = \dfrac{1 - xy}{1 + x^2y^2}$

39–40 Graph the function and observe where it is discontinuous. Then use the formula to explain what you have observed.

39. $f(x, y) = e^{1/(x - y)}$ **40.** $f(x, y) = \dfrac{1}{1 - x^2 - y^2}$

41–50 Determine the set of points at which the function is continuous.

41. $F(x, y) = \dfrac{xy}{1 + e^{x - y}}$ **42.** $F(x, y) = \cos\sqrt{1 + x - y}$

43. $F(x, y) = \dfrac{1 + x^2 + y^2}{1 - x^2 - y^2}$ **44.** $H(x, y) = \dfrac{e^x + e^y}{e^{xy} - 1}$

45. $G(x, y) = \sqrt{x} + \sqrt{1 - x^2 - y^2}$

46. $G(x, y) = \ln(1 + x - y)$

47. $f(x, y, z) = \arcsin(x^2 + y^2 + z^2)$

48. $f(x, y, z) = \sqrt{y - x^2} \ln z$

49. $f(x, y) = \begin{cases} \dfrac{x^2 y^3}{2x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 1 & \text{if } (x, y) = (0, 0) \end{cases}$

50. $f(x, y) = \begin{cases} \dfrac{xy}{x^2 + xy + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$

51–53 Use polar coordinates to find the limit. [If (r, θ) are polar coordinates of the point (x, y) with $r \geqslant 0$, note that $r \to 0^+$ as $(x, y) \to (0, 0)$.]

51. $\displaystyle\lim_{(x, y) \to (0, 0)} \frac{x^3 + y^3}{x^2 + y^2}$

52. $\displaystyle\lim_{(x, y) \to (0, 0)} (x^2 + y^2) \ln(x^2 + y^2)$

53. $\displaystyle\lim_{(x, y) \to (0, 0)} \frac{e^{-x^2 - y^2} - 1}{x^2 + y^2}$

54. Prove the three special limits in (2).

55. At the beginning of this section we considered the function

$$f(x, y) = \frac{\sin(x^2 + y^2)}{x^2 + y^2}$$

and guessed on the basis of numerical evidence that $f(x, y) \to 1$ as $(x, y) \to (0, 0)$. Use polar coordinates to confirm the value of the limit. Then graph the function.

56. Graph and discuss the continuity of the function

$$f(x, y) = \begin{cases} \dfrac{\sin xy}{xy} & \text{if } xy \neq 0 \\ 1 & \text{if } xy = 0 \end{cases}$$

57. Let

$$f(x, y) = \begin{cases} 0 & \text{if } y \leqslant 0 \quad \text{or} \quad y \geqslant x^4 \\ 1 & \text{if } 0 < y < x^4 \end{cases}$$

(a) Show that $f(x, y) \to 0$ as $(x, y) \to (0, 0)$ along any path through $(0, 0)$ of the form $y = mx^a$ with $0 < a < 4$.
(b) Despite part (a), show that f is discontinuous at $(0, 0)$.
(c) Show that f is discontinuous on two entire curves.

58. Show that the function f given by $f(\mathbf{x}) = |\mathbf{x}|$ is continuous on $\mathbb{R}^n$. [*Hint:* Consider $|\mathbf{x} - \mathbf{a}|^2 = (\mathbf{x} - \mathbf{a}) \cdot (\mathbf{x} - \mathbf{a})$.]

59. If $\mathbf{c} \in V_n$, show that the function f given by $f(\mathbf{x}) = \mathbf{c} \cdot \mathbf{x}$ is continuous on $\mathbb{R}^n$.

14.3 Partial Derivatives

Partial Derivatives of Functions of Two Variables

On a hot day, extreme humidity makes us think the temperature is higher than it really is, whereas in very dry air we perceive the temperature to be lower than the thermometer indicates. The National Weather Service has devised the *heat index* (also called the temperature-humidity index, or humidex, in some countries) to describe the combined effects of temperature and humidity. The heat index I is the perceived air temperature when the actual temperature is T and the relative humidity is H. So I is a function of T and H and we can write $I = f(T, H)$. The following table of values of I is an excerpt from a table compiled by the National Weather Service.

Table 1 Heat index *I* as a function of temperature and humidity

Relative humidity (%)

Actual temperature (°F): T \ H	50	55	60	65	70	75	80	85	90
90	96	98	100	103	106	109	112	115	119
92	100	103	105	108	112	115	119	123	128
94	104	107	111	114	118	122	127	132	137
96	109	113	116	121	125	130	135	141	146
98	114	118	123	127	133	138	144	150	157
100	119	124	129	135	141	147	154	161	168